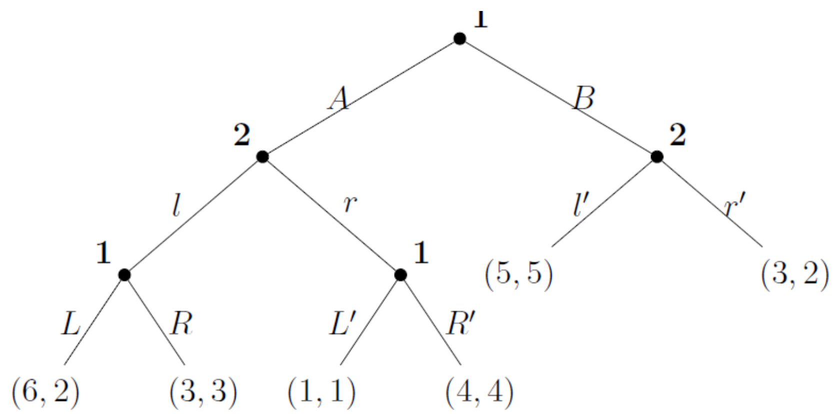
Micro III - August 2019 (Solution Guide)

1. Consider the following game G, where the first payoff is that of player 1, the second payoff that of player 2:



   (a) How many proper subgames are there in G (excluding the game itself)? What are the strategy sets of the players?

   SOLUTION: 4 proper subgames. $S_1 = \{A, B\} \times \{L, R\} \times \{L', R'\}$. $S_2 = \{l, r\} \times \{l', r'\}$.

   (b) Find all (pure strategy) Subgame-perfect Nash Equilibria in G.

   SOLUTION: Since perfect, complete information, we can solve by backward induction to get $SPNE = \{(B, L, R'), (r, l')\}$. *Both players earn a payoff of* 5.

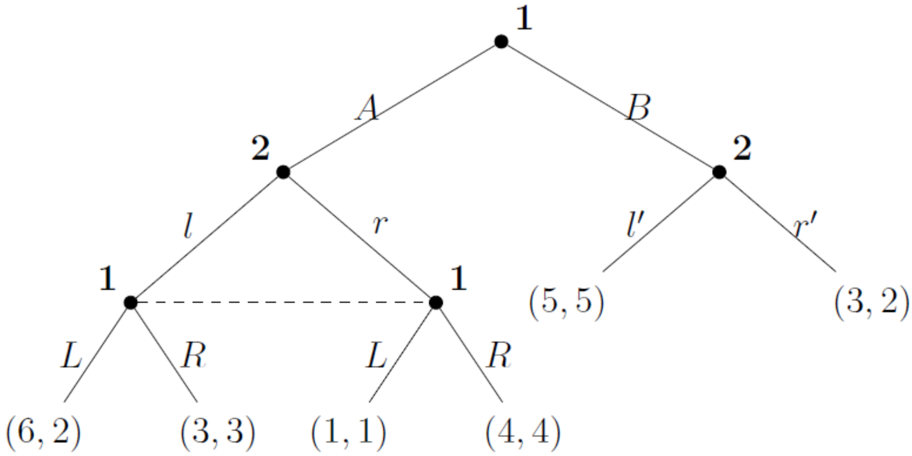
   (c) Suppose now that we modify G, so that player 1 does not observe the move of player 2 when he moves the second time. That is to say, player 1 does not observe whether player 2 chooses l or r.

      i. Draw the resulting game tree for the modified game.
      ii. Is this game of perfect or imperfect information? How many proper subgames are there (excluding the game itself)? What are the strategy sets of the players?
      iii. Show that there is a Subgame-Perfect Nash Equilibrium of the modified game where player 1 has a payoff of 6. Discuss briefly how player 1 benefits from not being able to observe player 2's action (max. 3 sentences).

   SOLUTION: See the figure below for the game tree. This is a game of imperfect information. There are 2 proper subgames. Player 2's strategy set is as before, but now $S_1 = \{A, B\} \times \{L, R\}$. *The subgame starting at player 2's choice between* l *and* r *can be written as:*

		Player 2	
		l	r
Player 1	L	$\underline{6}, \underline{2}$	$1, 1$
	R	$3, 3$	$\underline{4}, \underline{4}$

Thus, there are two NE of the subgame. We are looking for a SPNE with payoff 6 to player 1, so we take the NE (L, l). The right-hand side subgame is the same as before, with player 2 playing l'. Thus, player 1 has payoff 6 from playing A and payoff 5 from playing B. Therefore, $((A, L), (l, l'))$ is a SPNE and yields equilibrium payoff 6 to player 1. The unobservability of player 2's action implies that player 1 can credibly 'commit' to playing L if he believes that player 2 plays l. But this, in turn, makes it optimal for player 2 to play l. Player 2 could deviate by choosing action r, but in the modified game player 1 would not observe this, and thus continue to play L.



2. Two consumers are considering whether to buy a product that exhibits network effects. The payoff from buying depends on the choice of the other consumer. That is, for each consumer $i \in \{1, 2\}$, the payoff U_i from buying depends on three terms: the consumer's type, θ_i, which represents his intrinsic valuation of the product; a potential network payoff $\lambda > 0$, which consumer i only obtains if consumer $j \neq i$ also buys; and the price p. Specifically, buying yields $U_i = \theta_i + \lambda - p$ if consumer j also buys, or $U_i = \theta_i - p$ if consumer j does not. Not buying the product gives a payoff of zero. Each consumer's type is drawn from a uniform distribution on $[0, 1]$ and is private information. For all parts of this question, you can assume that $\lambda < p < 1$.

   Suppose consumers must simultaneously decide whether or not to buy, so the strategic situation they face can be seen as a static game of incomplete information.

   (a) Argue, in words, why the Bayes-Nash equilibrium of this game must be characterized by a threshold value of type, which we can denote by θ^*. That is, why is it that in equilibrium, a consumer with a high type, $\theta \geq \theta^*$, will buy the product, but a consumer with a low type, $\theta < \theta^*$, will not?

   SOLUTION: the key point here is that willingness to pay is increasing in type, θ. Thus, there cannot exist a BNE where a consumer of type θ' buys and a consumer of type $\theta'' > \theta'$ does not; because buying would give a higher payoff to θ'' than it would to θ'. In such a candidate equilibrium, at least one consumer type would have a profitable deviation: either type $\theta = \theta''$ would deviate by buying, or type $\theta = \theta'$ would deviate by taking her outside option. Thus, any BNE must be characterized by a threshold value of type.

(b) Suppose that consumer j's strategy is to buy the product if and only if $\theta_j \geq \theta^*$. Show that consumer i will find it optimal to buy, given consumer j's strategy, if and only if $\theta_i \geq p - \lambda(1-\theta^*)$.

SOLUTION: Consumer i's expected payoff from buying, given consumer j's strategy, is $\theta_i + P(\theta_j \geq \theta^*) - p = \theta_i + \lambda(1-\theta^*) - p$. *Thus, consumer i will find it optimal to buy if and only if* $\theta_i + \lambda(1-\theta^*) - p \geq 0$, *which is equivalent to* $\theta_i \geq p - \lambda(1-\theta^*)$.

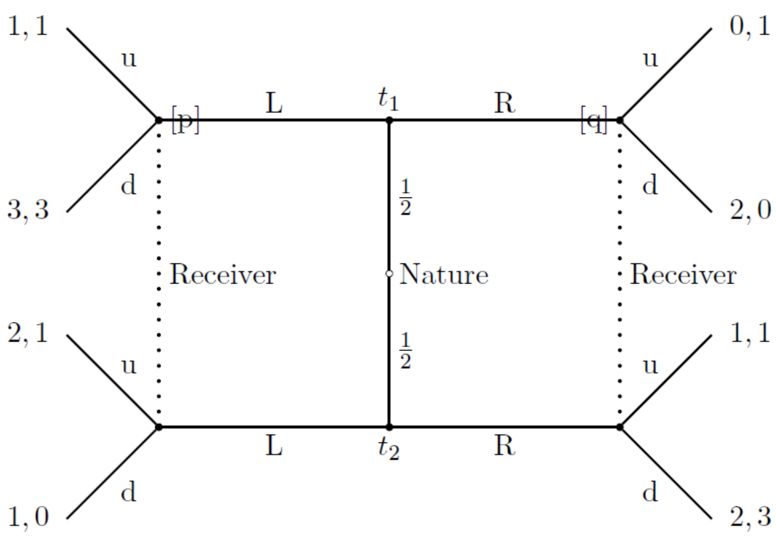
(c) Using your answer in part (b), solve for the value of θ^* in the Bayes-Nash equilibrium of this game. Explicitly write down the consumers' equilibrium strategies.

SOLUTION: In equilibrium, type $\theta_i = \theta^*$ *should be indifferent about buying. That is,* $\theta^* = p - \lambda(1-\theta^*)$, *or equivalently* $\theta^* = \frac{p-\lambda}{1-\lambda}$. *Thus, the equilibrium strategy of consumer i is to buy if and only if* $\theta_i \geq \frac{p-\lambda}{1-\lambda}$. *The equilibrium strategy of consumer j is to buy if and only if* $\theta_j \geq \frac{p-\lambda}{1-\lambda}$.

(d) Using your answer in part (c), show how a change in λ will affect the probability that consumers will buy the product. What is the intuition for this result?

SOLUTION: The probability of buying in equilibrium is $P(\theta_j \geq \theta^*) = P(\theta_j \geq \frac{p-\lambda}{1-\lambda}) = 1 - \frac{p-\lambda}{1-\lambda} = \frac{1-p}{1-\lambda}$. *Note that* $\lambda < p < 1$ *implies that this probability lies between 0 and 1. The derivative of* $P(\theta_j \geq \theta^*)$ *with respect to* λ *is* $\frac{1-p}{(1-\lambda)^2}$, *which is strictly positive. Thus, an increase in* λ *leads to a higher equilibrium probability of buying. There are two effects at play, both of which push in the same direction. First, a stronger network effect will increase a consumer's expected payoff from buying, holding the strategy of the other consumer constant. This direct effect pushes up both consumers' probability of buying. Second, given that each consumer realizes the other is now more likely to buy, this increases her own expected payoff from buying even more. This indirect effect pushes up both consumers' probability of buying further still.*

3. Now consider the following game G':



(a) Is G' a repeated game? Briefly explain your answer.

SOLUTION: By definition, G' is a dynamic game, but it is not a repeated game, because there is no stage game repeated multiple times.

(b) Find two pooling equilibria in G': one where both sender types play L, and another where both sender types play R. Do they satisfy Signaling Requirement 6 ('equilibrium domination')?

SOLUTION: Pooling equilibria: $(LL, du, p = 1/2, q \geq 2/3)$ *and* $(RR, ud, p \leq 1/3, q = 1/2)$. *Pooling on R satisfies Signaling Requirement 6. Pooling on L does not; this requirement imposes* $q = 0$, *since the message R for type 1 is equilibrium dominated by L.*

(c) Find all separating equilibria in G'.

SOLUTION: Separating equilibrium: $(LR, dd, p = 1, q = 0)$

(d) Which of the equilibria you found in parts (b) and (c) seems most reasonable? Explain your answer briefly using concepts from the course (2-3 sentences).

SOLUTION: Pooling on R might seem more reasonable than pooling on L, since the latter does not satisfy Signaling Requirement 6. One could also argue that the separating equilibrium is the most reasonable of all, since it is Pareto dominant.